

Irrationality of the special values of formal Laurent series represented by the formal Mellin transform of G -functions

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In this talk, we introduce the formal Mellin transform \mathcal{M} as follows:

$$\mathcal{M} : \overline{\mathbb{Q}}[[z]] \longrightarrow \frac{1}{z} \overline{\mathbb{Q}} \left[\left[\frac{1}{z} \right] \right], \quad g(z) \mapsto \mathcal{M}(g)(z) := \sum_{k=0}^{\infty} b_k \left(\frac{-1}{z} \right)^{k+1}$$

where $\{b_k\}_{k \in \mathbb{Z}_{\geq 0}}$ are defined by $g(e^z - 1) = \sum_{k=0}^{\infty} \frac{b_k}{k!} z^k$. Then we obtain the irrationality results of special values of the formal Laurent series which is the image of formal Mellin transform as follows:

Proposition.— *Let p be a prime number. Let $g(z) = \sum_{k=0}^{\infty} a_k z^k \in \overline{\mathbb{Q}}[[z]]$ is a G -function satisfying*

- (i) *There exists $\delta > 0$ and $C_1 > 0$ satisfying $|a_j| \leq \delta C_1^j$ for all $j \in \mathbb{Z}_{\geq 0}$,*
- (ii) *For any $\epsilon > 0$, there exists $\gamma = \gamma(\epsilon) > 0$ and $C_2 \geq 1$ satisfying*
 $\text{den}(a_j) \leq \gamma C_2^{j(1+\epsilon)}$ *for all $j \in \mathbb{Z}_{\geq 0}$,*
- (iii) *Put r is the p -adic radius of convergence of $g(z)$. Then $r \geq 1$,*
- (iv) *$g(z) \notin \overline{\mathbb{Q}}[\log(1+z), (1+z)^\alpha \mid \alpha \in \overline{\mathbb{Q}}]$.*

Let $\alpha \in \{\alpha \in \overline{\mathbb{Q}} \mid |\alpha|_p > 1\}$ satisfying

$$rp^{\frac{2}{p-1}} |\alpha|_p^2 > e \cdot \text{den}(\alpha) \prod_{q \mid \text{den}(\alpha), q: \text{prime}} q^{\frac{1}{q-1}} \times \begin{cases} 16(C_1 C_2)^3 & \text{if } 1 < C_1, \\ 16C_1^2 C_2^3 & \text{if } \frac{1}{2} < C_1 \leq 1, \\ 4C_2^3 & \text{if } C_1 \leq \frac{1}{2}. \end{cases}$$

Then $\mathcal{M}(g)(\alpha)$ is an element of $\mathbb{Q}_p \setminus \mathbb{Q}$.

By using Proposition, we prove the irrationality of special values of p -adic Hurwitz zeta function. Proposition proved by the theory of Padé approximation. In this talk, we also mention a generalization of Proposition which give a lower bound of the dimension of the vector space spanned by the special values of p -adic Hurwitz zeta function and other some functions.